Vector Algebra

Vector: vector is a physical quantity which has both magnitude and direction.

A is the initial point and B is the terminal point. When initial and terminal point will be same, then this vector is the zero vector $\vec{0}$.

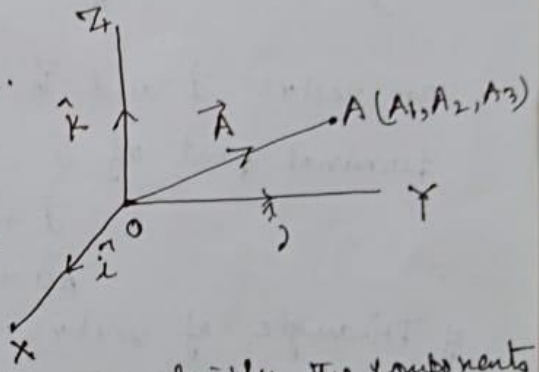
Magnitude of the vector $\vec{AB}$ is denoted by $|\vec{AB}|$.

Unit vector: whose magnitude is 1.

Unit vector in the direction of $\vec{AB} = \frac{\vec{AB}}{|\vec{AB}|}$

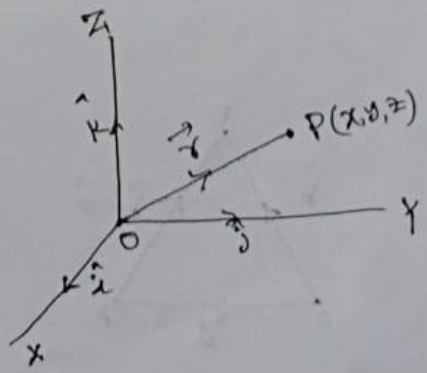
Components of a vector:

If O be the initial point and $A(A_1, A_2, A_3)$ be point in 3-d space. Then $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ is the vector $\vec{OA}$; where $A_1\hat{i}$, $A_2\hat{k}$, $A_3\hat{k}$ are the rectangular component vectors ~~are also known as~~ or simply component vectors in the x, Y, Z directions. Again A_1, A_2, A_3 are simply the components of $\vec{A}$ in x, Y, Z directions.



position vector: position vector or Radius vector is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$



Length of a vector (Magnitude):

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Then $|\vec{r}|^2 = \vec{r} \cdot \vec{r}$

$= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k})$

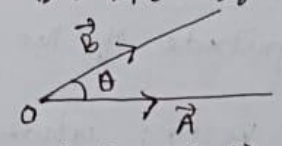
$= x^2 + y^2 + z^2$

$\therefore |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

Dot product (Scalar product):

$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

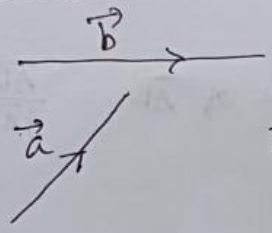
$\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$.

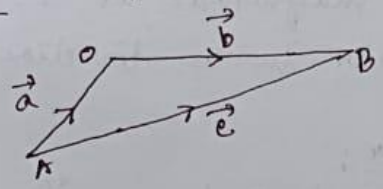


$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

where $0 \leq \theta \leq \pi$

Addition of two vectors:



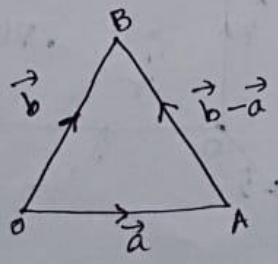


Two vectors $\vec{a}$ and $\vec{b}$ acting at a point O. That is terminal point of $\vec{a}$ is the initial point of $\vec{b}$. Then

$\vec{a} + \vec{b} = \vec{c}$

$\vec{AO} + \vec{OB} = \vec{AB}$. This is known as Law of Triangle of vectors.

Subtraction of two vectors: Let $\vec{a}$ and $\vec{b}$ be two vectors. Then $\vec{a} + (-\vec{b}) = \vec{a} - \vec{b}$ is the subtraction.



By the Law of triangle of vectors,

$\vec{OA} + \vec{AB} = \vec{OB}$

$\therefore \vec{AB} = \vec{OB} - \vec{OA}$

$= \vec{b} - \vec{a}$.